Topics in Biophysics   Graduate Seminar class

I. Molecular interactions.

Forces which arise from fluctuations (all forces?)
e.g. entropic forces are particularly important in bio-molecular systems → we start with those.

Simplest "entropic force" is the pressure of an ideal gas: states with different volumes V have the same energy E, but entropy increases with $\ln V$:

$S \propto \ln V$ so increasing the volume lowers the free energy $F = E - TS$

→ pressure $p = -\dfrac{\partial F}{\partial V} = T\dfrac{\partial S}{\partial V} \propto \dfrac{T}{V}$

→ force on the walls of the container.

General features of forces in bio-molecular systems:

1. often the important forces are entropic in origin (e.g. hydrophobic interaction)
2. dynamics is always overdamped (no oscillations) (forces are "small" and dissipation is "large")

at fixed T, i.e. consider the situation:

then E is indep. of V (depends only on T: $E = \frac{3}{2} NkT$)

To get a feeling for overdamped dynamics :

<u>examples</u> a) 1 μm bacterium (E. coli)

mass $m \simeq 10^{-12}$ g

swimming at $v = 10\ \mu m/s$. If the bacterium stops swimming, the drift velocity goes to zero with a charact. time $\tau \sim \frac{m}{\eta r}$, $\eta \simeq 10^{-2}$ g/cm s viscosity of water

(by dim. anal., or Stokes law etc.) [exact: $\tau = \frac{m}{6\pi\eta r}$]

so $\tau \sim \frac{10^{-12}}{10^{-2} \times 10^{-4}} = 10^{-6}$ s $= 1\ \mu s$ (Dimensions of η)

During this time, the bacterium moves by $v\tau \simeq 10^{-11}$ m $= 0.1$ Å !

I.e. the bacterium stops dead the minute it stops swimming. Except for thermal noise !

$\rightarrow$ eq. of motion is typically a Langevin eq. :

$F \propto v$ not $F \propto \dot{v}$: $\dot{x} = \mu F + \Gamma(t)$

$\Gamma(t)$ stochastic function (thermal noise) defined by its spectrum (or correlations); typically one takes

$\langle \Gamma(t)\, \Gamma(t+\tau) \rangle \propto \delta(\tau)$

$\longrightarrow$

Overdamping : $\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$

$x(t) = A e^{i\omega t} \quad \Rightarrow \quad -\omega^2 + i\gamma\omega + \omega_0^2 = 0$

$\Rightarrow \quad \omega = i\frac{\gamma}{2} \pm \sqrt{-\frac{\gamma^2}{4} + \omega_0^2}$

if $-\frac{\gamma^2}{4} + \omega_0^2 < 0$ then ω is pure imag.

$\rightarrow$ overdamped motion. So $\gamma > 2\omega_0 \rightarrow$ overdamped.

In terms of the spring const. K : $\omega_0^2 = \frac{K}{m}$

so $\gamma > 2\sqrt{\frac{K}{m}} \rightarrow$ overdamped.

Relevant parameter : $\frac{\gamma^2 m}{K}$. Actually, $\gamma = \frac{\Gamma}{m}$ with Γ indep. of mass, so the parameter is really $\frac{\Gamma}{\sqrt{Km}}$. To avoid confusion, start with:

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \omega_0^2 x = 0$$

$$\omega_0^2 = \frac{K}{m}.$$

$m\ddot{x} = -6\pi\eta R\dot{x} \quad \Rightarrow \quad \dot{u} = -\frac{6\pi\eta R}{m} u$

$\Rightarrow \quad u = u(0)e^{-t/\tau}, \quad \tau = \frac{m}{6\pi\eta R}.$

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \frac{K}{m}x = 0$$

$$\left[\frac{\gamma}{m}\right] = \frac{1}{t} \qquad \text{overdamped if} \quad \frac{\gamma^2}{mK} > 1$$

$$\left[\frac{K}{m}\right] = \frac{1}{t^2}$$

Bead : $\gamma = 6\pi\eta R$, $K = \dfrac{kT}{l_p^2 N}$

b) polymer spring holding a μm size particle :

$$K = \frac{kT}{l_p^2 N} \quad \text{entropic spring const.}$$
(see later) →

$\gamma = 6\pi\eta r/m$; $\quad m = \frac{4}{3}\pi r^3 \rho \qquad \Gamma = 6\pi\eta r$

say $r = 5$ μm ; taking "most unfavorable" values

$l_p = 1$ μm

$N = 10$

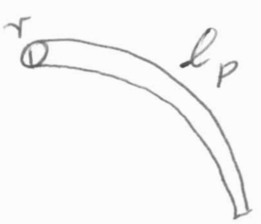
$$\frac{\Gamma^2}{Km} \sim \frac{\eta^2 N l_p^2}{kT\rho r} \sim \frac{10^{-4} \times 10 \times 10^{-14}}{5\times 10^{-7} \times 1 \times 10^{-14}}$$

($kT \simeq 4\times 10^{-14}$ ergs at room temp. ; also :

$\frac{kT}{1\,\mu m} \simeq 4$ pN $\longrightarrow$ 1 pN good scale for forces at bio-molecular level)

so $\frac{\gamma^2}{mK} \sim 10^{4}$!!

c) long-wavelength bending modes of polymer =



cylinder of radius r, length l_p

elastic ~~modulus~~ spring const. :

elastic energy per unit length (bent rod) is $\frac{E}{l} = \frac{1}{2} B \frac{1}{R^2}$, B bending modulus

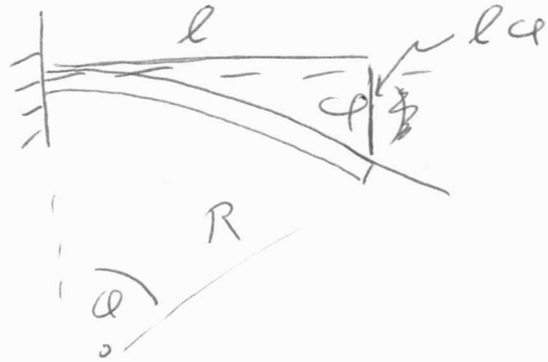
(Hooke's law) $\qquad R$ radius of curvature

relation to the persistence length :

$\frac{B}{l_p^2} \sim \frac{kT}{l_p} \Rightarrow B \sim kT\, l_p$ [exact:

$[B]$ = energy × length

rod :



$L = I\dot{\varphi}$, $I\ddot{\varphi} = \dot{L}$ = torque

$= \frac{dE}{d\varphi}$ E elastic energy

now $l^2 + (l\varphi)^2 \simeq (R\varphi)^2$ $\varphi \ll 1$

$\Rightarrow \frac{1}{R^2} \simeq \frac{\varphi^2}{l^2}$

$E = \frac{1}{2}\frac{B}{R^2}\, l$

$= \frac{1}{2} B \frac{\varphi^2}{l}$

$\Rightarrow \frac{dE}{d\varphi} = \frac{B}{l}\varphi$

and $I\ddot{\varphi} - \frac{B}{l}\varphi = 0$ eq. of motion

so "$\frac{k}{m}$" $= \frac{B}{Il}$; $I \propto m l^2 \Rightarrow k \sim \frac{B}{l^3} = \frac{kT}{l_p^2}$

The drag force is $\sim \eta\, l_p\, u$ so "Γ" $= \eta\, l_p$

[drag on a cylinder of length l, radius r, see Landau:

$F_d = \frac{4\pi \eta l u}{\ln(3.7\,\nu / u r)}$]

$m = r^2 l_p \rho$

$\Rightarrow \frac{\gamma^2}{mk} \sim \frac{\eta^2 l_p^2\, l_p^2}{r^2 l_p \rho\, kT} = \left(\frac{l_p}{r}\right)^2 \frac{\eta^2 l_p}{\rho\, kT}$

e.g. ds DNA : $r = 1$ nm, $l_p \simeq 50$ nm

$\Rightarrow \frac{\gamma^2}{mk} \sim 10^7$!!